HA S. 145/7 ①

geg.: $f_a(x) = -\frac{1}{2a^2}x^4 + \frac{1}{a}x^3$; $a \in \mathbb{R}^+$; K_a

ges.: a) Nst., P_y, Ext., P_W, K_2 in][-2|4]

Lös.: Nst:

$P_y(0|0)$ ✓

$0 = -\frac{1}{2a^2}x^4 + \frac{1}{a}x^3$

$0 = x^3 \cdot \left(-\frac{1}{2a^2}x + \frac{1}{a}\right)$

$x_{0_1} = 0$ ✓

$0 = -\frac{1}{2a^2}x + \frac{1}{a}$

$\frac{1}{2a^2}x = \frac{1}{a}$

$x_{0_2} = 2a$ ✓

③

Ext:

$f_a'(x) = -\frac{2}{a^2}x^3 + \frac{3}{a}x^2$ ✓

nB: $0 = x^2\left(-\frac{2}{a^2}x + \frac{3}{a}\right)$

$x_{E_1} = 0$ ✓

$0 = -\frac{2}{a^2}x + \frac{3}{a}$

$\frac{2}{a^2}x = \frac{3}{a} \wedge x_{E_2} = \frac{3}{2}a$ ✓

⑦

$f_a''(x) = -\frac{6}{a^2}x^2 + \frac{6}{a}x$ ✓

h.B. $f_a''(0) = 0 \wedge$ keine Extremstelle, evtl. P_W ✓

$f_a''\left(\frac{3}{2}a\right) = -\frac{6}{a^2} \cdot \frac{9}{4}a^2 + \frac{6}{a} \cdot \frac{3}{2}a = -\frac{27}{2} + 9 < 0$ (-4,5) ✓

$\wedge$ Max $P_H\left(\frac{3}{2}a \,\middle|\, \frac{27}{32}a^2\right)$ ✓

WP:

nB. $0 = -\frac{6}{a^2}x^2 + \frac{6}{a}x = x\left(-\frac{6}{a^2}x + \frac{6}{a}\right)$

$x_W = 0$ ✓ $\quad 0 = -\frac{6}{a^2}x + \frac{6}{a}$

$\frac{6}{a^2}x = \frac{6}{a} \Leftrightarrow \underline{\underline{x_w = a}}$

$f_a'''(x) = -\frac{12}{a^2}x + \frac{6}{a}$

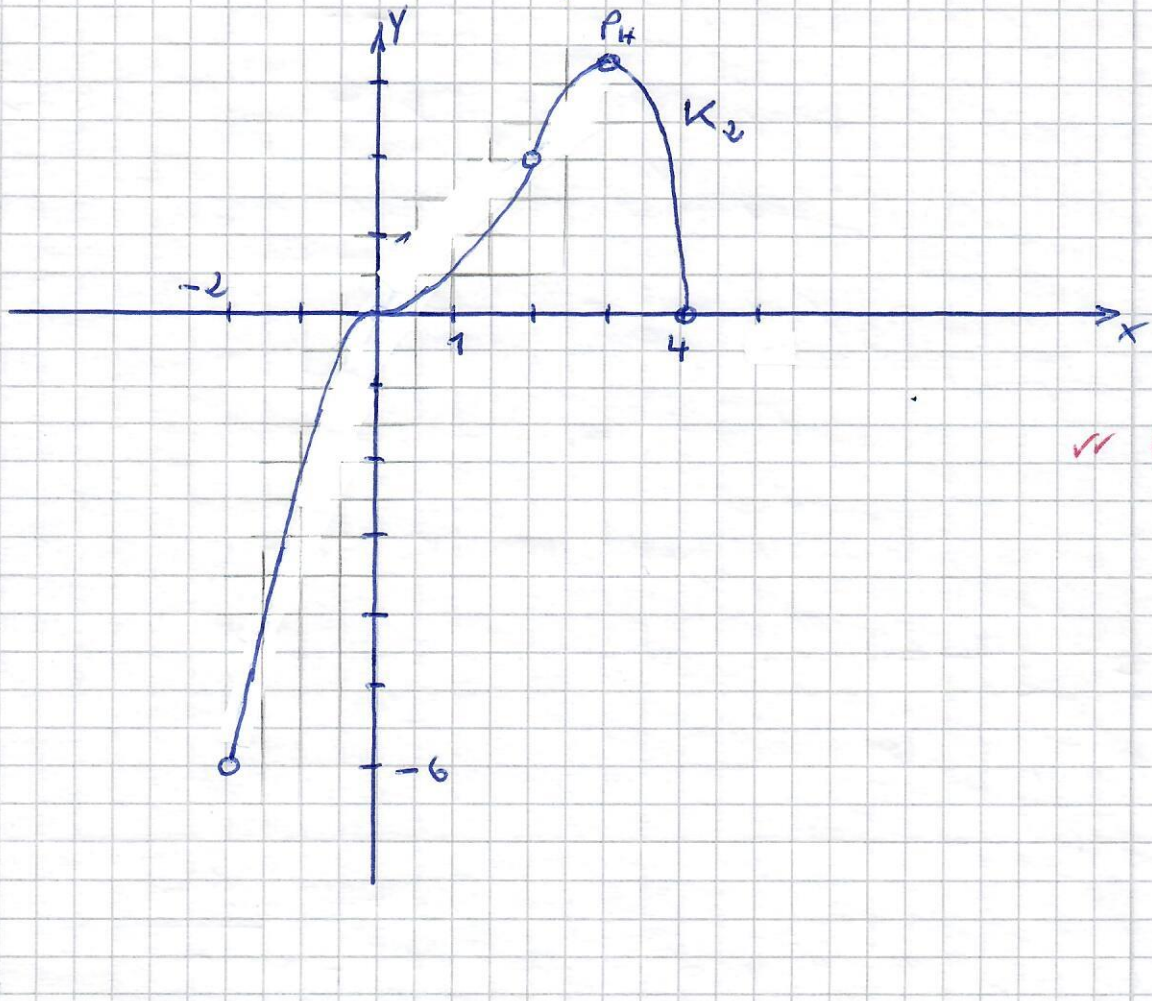
h.B. $f_a'''(0) = \frac{6}{a} > 0$ da $a > 0$ $\Leftrightarrow$ ReLiKu

$f_a'''(a) = -\frac{12}{a} + \frac{6}{a} = -\frac{6}{a} < 0$ da $a > 0$

$\Leftrightarrow$ LiReKu

$\underline{\underline{\Leftrightarrow P_{w_1}(0|0);\ P_{w_2}\left(a\middle|\tfrac{1}{2}a^2\right)}}$ 6

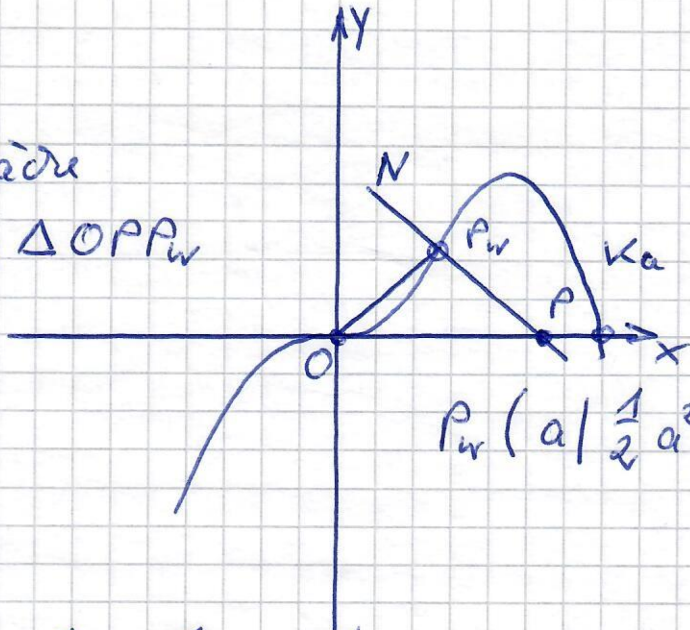
Daten für K_2:		
	Nst	$x_{0_1} = 0;\ x_{0_2} = 4$
	P_Y	$P_Y(0|0)$
	Ex	$P_H\left(3\middle|\frac{27}{8}\right)$
	WP	$P_{w_1}(0|0);\ P_{w_2}(2|2)$



2

b) Skizze:

ges.: Fläche des ΔOPP_w



$P_w\left(a \mid \frac{1}{2}a^2\right)$

Lösung: $A = \frac{1}{2} g \cdot h_g$

$h_g = \frac{1}{2}a^2$

g = Nullstelle der Normalen x_P

NR: $f_a'(x_{P_w}) = -\frac{2}{a^2}(a)^3 + \frac{3}{a}a^2 = a = m_t$

$m_N = -\frac{1}{m_t} = \underline{-\frac{1}{a}}$ ✓

$\wedge N$: $y = -\frac{1}{a}x + n$ halbfertige N-Gl.

$P_w \rightarrow$ $\frac{1}{2}a^2 = -\frac{1}{a} \cdot a + n$

$n = \frac{1}{2}a^2 + 1$

$\wedge N$: $\underline{\underline{y = -\frac{1}{a}x + \frac{1}{2}a^2 + 1}}$ fertige N-Gl. ✓

Nullstelle:

$0 = -\frac{1}{a}x + \frac{1}{2}a^2 + 1$

$\frac{1}{a}x = \frac{1}{2}a^2 + 1 \quad | \cdot a$ 5

$\underline{\underline{x_0 = \frac{1}{2}a^3 + a = g}}$ ✓

A(a) = 13 ✓
$\wedge a \approx 2{,}38$
CP!

$A(a) = \frac{1}{2} \cdot \left(\frac{1}{2}a^3 + a\right) \cdot \frac{1}{2}a^2 = \underline{\underline{\frac{1}{4}a^2\left(\frac{1}{2}a^3 + a\right)}}$ ✓

RF 20