

S. 122/4

①

$$\begin{aligned} \underline{e)} \quad & 2r - s + at = 0 \\ & 3r + 3s + 3t = 0 \\ & 5r + 6s + 2t = 0 \end{aligned}$$

$$\begin{array}{cccc} 2 & -1 & a & 0 \\ 3 & 3 & 3 & 0 \\ 5 & 6 & 2 & 0 \end{array}$$

$$\begin{array}{l} \downarrow \cdot 3 \\ \downarrow \cdot 2 \end{array} \quad \begin{array}{l} \downarrow \cdot 5 \\ \downarrow \cdot 2 \end{array}$$

$$\begin{array}{cccc} 2 & -1 & a & 0 \\ 0 & -9 & 3a-6 & 0 \\ 0 & -17 & 5a-4 & 0 \end{array}$$

$$\downarrow \begin{array}{l} \cdot 17 \\ \cdot 9 \end{array}$$

$$\begin{array}{cccc} 2 & -1 & a & 0 \\ 0 & -9 & 3a-6 & 0 \\ 0 & 0 & * & 0 \end{array}$$

$$* \quad \underbrace{(17(3a-6) - 9(5a-4))}_{=0 \text{ für lin. abh.}} t = 0$$

$$\rightarrow 0 = 51a - 102 - 45a + 36 = 0$$

$$6a = 66$$

$$\underline{\underline{a = 11}}$$

$$\underline{j)} \quad \begin{array}{cccc} 1 & 2 & 1 & 0 \\ a & 8 & 1 & 0 \\ a^2 & 18 & 1 & 0 \end{array}$$

$$\downarrow \cdot a \quad \downarrow \cdot a^2$$

$$\begin{array}{cccc} 1 & 2 & 1 & 0 \\ 0 & 2a-8 & a-1 & 0 \\ 0 & 2a^2-18 & a^2-1 & 0 \end{array}$$

$$\downarrow \begin{array}{l} \cdot 2a^2-18 \\ \cdot 2a-8 \end{array}$$

$$\begin{array}{cccc} 1 & 2 & 1 & 0 \\ 0 & 2a-8 & a-1 & 0 \\ 0 & 0 & * & 0 \end{array}$$

$$* \quad \underbrace{((2a^2-18)(a-1) - (2a-8)(a^2-1))}_{=0 \text{ für lin. abh.}} t = 0$$

$$\rightarrow \cancel{2a^3} - 2a^2 - 18a + 18 - \cancel{2a^3} + 2a + 8a^2 - 8 = 0$$

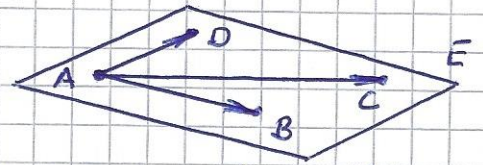
$$6a^2 - 16a + 10 = 0$$

$$\text{GTR} \left\{ \begin{array}{l} a_1 = 1 \\ a_2 = \frac{5}{3} \end{array} \right.$$

a)

$$r \cdot \vec{AB} + s \cdot \vec{AC} + t \cdot \vec{AD} = \vec{0}$$

hat ∞ viele Lösungen,
einschließlich der trivialen, wenn A, B, C, D auf E liegen.



Prüfen:

$$r \cdot \begin{pmatrix} 1 \\ 9 \\ 7 \end{pmatrix} + s \cdot \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + t \cdot \begin{pmatrix} -4 \\ 15 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{l} \text{I} \\ \text{II} \\ \text{III} \end{array} \quad \begin{array}{l} r \\ 9r \\ 7r \end{array} + \begin{array}{l} + 2s \\ + s \\ + 3s \end{array} - \begin{array}{l} 4t \\ + 15t \\ + 5t \end{array} = \begin{array}{l} 0 \\ 0 \\ 0 \end{array}$$

$$\begin{array}{l} A_2 \\ \\ \\ \end{array} \quad \begin{array}{cccc} 1 & 2 & -4 & 0 \\ 9 & 1 & 15 & 0 \\ 7 & 3 & 5 & 0 \end{array} \quad \begin{array}{l} \downarrow + \cdot (-9) \\ \downarrow + \cdot (-7) \end{array}$$

$$\begin{array}{cccc} 1 & 2 & -4 & 0 \\ 0 & -17 & 51 & 0 \\ 0 & -11 & 33 & 0 \end{array} \quad \begin{array}{l} : 17 \\ : 11 \end{array}$$

$$\begin{array}{l} A_2 \\ \\ \\ \end{array} \quad \begin{array}{cccc} 1 & 2 & -4 & 0 \\ 0 & -1 & 3 & 0 \\ 0 & -1 & 3 & 0 \end{array} \quad \downarrow -$$

$$\begin{array}{cccc} 1 & 2 & -4 & 0 \\ 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}$$

$$\Rightarrow 0 \cdot t = 0 \Rightarrow t = R \Rightarrow \text{lin. abh.} \\ \Rightarrow \underline{\underline{A, B, C, D \in E}}$$

b) Prüfen:

$$r \cdot \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} + s \cdot \begin{pmatrix} -12 \\ 3 \\ 4 \end{pmatrix} + t \cdot \begin{pmatrix} -1 \\ 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

mit CP: $L = \{(0|0|0)\} \Rightarrow$ nur triviale Los.

$\Rightarrow$ lin. unabh.

$\Rightarrow \underline{\underline{A, B, C, D \text{ nicht alle } \in E}}$

LOS:

4

c) I) $4x - 8y + 2z = 1$

II) $-2x - 2y + 4z = -1$

$$\begin{array}{cccc} 4 & -8 & 2 & 1 \\ -2 & -2 & 4 & -1 \end{array} \quad \downarrow + \cdot 2$$

$$\begin{array}{cccc} 4 & -8 & 2 & 1 \\ 0 & -12 & 10 & -1 \end{array} \quad z \in \mathbb{R}, z \text{ ist Parameter}$$

$$\begin{array}{rcl} 4x - 8y & = & 1 - 2z \\ -12y & = & -1 - 10z \end{array}$$

$$-12y = -1 - 10z \quad | : (-12)$$

$$y = \frac{1}{12} + \frac{10}{12}z \rightarrow \dots$$

$$4x - 8\left(\frac{1}{12} + \frac{10}{12}z\right) = 1 - 2z$$

$$4x - \frac{2}{3} - \frac{80}{12}z = 1 - 2z$$

$$4x = 1 + \frac{2}{3} - 2z + \frac{20}{3}z$$

$$4x = 1 + \frac{2}{3} - 2z + \frac{20}{3}z$$

$$4x = \frac{5}{3} + \frac{14}{3}z \quad | : 4$$

$$x = \frac{5}{12} + \frac{14}{12}z$$

$$\hookrightarrow L = \left\{ (x|y|z) \mid \left(\frac{5}{12} + \frac{14}{12}z \mid \frac{1}{12} + \frac{10}{12}z \mid z \right) \right\}$$

d) I) $-x + 2y + z = 4$

II) $3x + y - 4z = 1$

$$\begin{array}{rcl} -x + 2y + z & = & 4 \\ 3x + y - 4z & = & 1 \end{array} \quad \downarrow + \cdot 3$$

$$\begin{array}{rcl} -x + 2y + z & = & 4 \\ 7y - 3z & = & 13 \end{array}$$

$$\begin{array}{rcl} -x + 2y + z & = & 4 \\ -3z & = & 13 - 7y \end{array}$$

$$z = -13 + 7y$$

$y \in \mathbb{R}$, y als Parameter

$$-x - 13 + 7y = 4 - 2y$$

$$\begin{array}{rcl} -x & = & 4 + 13 - 2y - 7y \\ x & = & -17 + 9y \end{array}$$

$$\hookrightarrow L = \left\{ (x|y|z) \mid (-17 + 9y | y | -13 + 7y) \right\}$$