LB

S. 355/2

a) $E_1: \left(\vec{x} - \begin{pmatrix}1\\2\\0\end{pmatrix}\right) \circ \begin{pmatrix}5\\0\\1\end{pmatrix} = 0$; $E_2: \left(\vec{x} - \begin{pmatrix}2\\3\\7\end{pmatrix}\right) \circ \begin{pmatrix}6\\1\\0\end{pmatrix} = 0$

$\sphericalangle E_1, E_2$:

$$\cos\sphericalangle(E_1, E_2) = \frac{|\vec{n}_1 \circ \vec{n}_2|}{|\vec{n}_1| \cdot |\vec{n}_2|} = \frac{\left|\begin{pmatrix}5\\0\\1\end{pmatrix} \circ \begin{pmatrix}6\\1\\0\end{pmatrix}\right|}{\sqrt{25+1} \cdot \sqrt{36+1}}$$

$$\cos\sphericalangle(E_1, E_2) = \frac{|30|}{\sqrt{26} \cdot \sqrt{37}} = 0{,}9672$$

$$\sphericalangle(E_1, E_2) = \arccos(0{,}9672) \approx \underline{\underline{14{,}7^\circ}}$$

bzw. CP: $\text{angle}(\vec{n}_1, \vec{n}_2) = \text{angle}\left(\begin{pmatrix}5\\0\\1\end{pmatrix}, \begin{pmatrix}6\\1\\0\end{pmatrix}\right) = \underline{\underline{14{,}7^\circ}}$

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b) $\vec{n}_1$ von E_1: $\vec{n}_1 = \begin{pmatrix}1\\1\\1\end{pmatrix}$; $\vec{n}_2$ von E_2: $\vec{n}_2 = \begin{pmatrix}1\\-1\\7\end{pmatrix}$

CP: $\sphericalangle(E_1, E_2) = \text{angle}(\vec{n}_1, \vec{n}_2) = \text{angle}\left(\begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\-1\\7\end{pmatrix}\right) = \underline{\underline{55{,}5^\circ}}$

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c) $\vec{n}_1 = \begin{pmatrix}3\\5\\0\end{pmatrix}$; $\vec{n}_2 = \begin{pmatrix}2\\-3\\-3\end{pmatrix}$

CP: $\sphericalangle(E_1, E_2) = \text{angle}\left(\begin{pmatrix}3\\5\\0\end{pmatrix}, \begin{pmatrix}2\\-3\\-3\end{pmatrix}\right) = 109{,}2^\circ \curvearrowright \underline{\underline{70{,}8^\circ}}$

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d) $\vec{n}_1 = \begin{pmatrix}0\\1\\0\end{pmatrix}$; $\vec{n}_2 = \begin{pmatrix}1\\0\\0\end{pmatrix}$

CP: $\sphericalangle(E_1, E_2) = \underline{\underline{90^\circ}}$ aus Anschauung

S. 256/3

a) $g: \vec{x} = \begin{pmatrix} 1 \\ 4 \\ 9 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$; $E: 3x + 5y - 2z = 7$

$\sphericalangle g, E$: $\sin \sphericalangle(g, E) = \dfrac{|\vec{a} \circ \vec{n}|}{|\vec{a}| \cdot |\vec{n}|} = \dfrac{\left| \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \circ \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix} \right|}{\sqrt{1+4+1} \cdot \sqrt{9+25+4}}$

$\sin \sphericalangle(g, E) = \dfrac{|11|}{\sqrt{6} \cdot \sqrt{38}} \approx 0{,}7285$

$\sphericalangle(g, E) = \arcsin(0{,}7285) \approx \underline{\underline{46{,}8^\circ}}$

bzw.: CP: $\text{angle}(\vec{a}, \vec{n}) = \text{angle}\left(\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix} \right) \approx 43{,}2^\circ$

$\curvearrowright \sphericalangle(g, E) = 90^\circ - 43{,}2^\circ = \underline{\underline{46{,}8^\circ}}$

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b) $\vec{a}$ von g: $\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$; $\vec{n}$ von E: $\vec{n} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

CP: $\sphericalangle(\vec{a}, \vec{n}) = \text{angle}\left(\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \right) = 0^\circ$

$\curvearrowright \sphericalangle(g, E) = 90^\circ - 0^\circ = \underline{\underline{90^\circ}}$

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c) $\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$; $\vec{n} = \begin{pmatrix} 6 \\ -3 \\ 0 \end{pmatrix}$

CP: $\sphericalangle(\vec{a}, \vec{n}) = \text{angle}\left(\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 6 \\ -3 \\ 0 \end{pmatrix} \right) = 90^\circ$

$\curvearrowright \sphericalangle(g, E) = 90^\circ - 90^\circ = \underline{\underline{0^\circ}} \curvearrowright g \parallel E$

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d) $\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$; $\vec{n} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$

CP: $\sphericalangle(\vec{a}, \vec{n}) = \text{angle}\left(\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \right) = 90^\circ$

$\curvearrowright \sphericalangle(g, E) = 90^\circ - 90^\circ = \underline{\underline{0^\circ}} \curvearrowright g \parallel E$

S. 356/4

a) $\sphericalangle(z\text{-Achse}, E)$

Gleichung der z-Achse: $\vec{x} = z\vec{k} = z\begin{pmatrix}0\\0\\1\end{pmatrix}$

$E: 2x + z = -4$ // y-Achse!

$\Rightarrow \quad \vec{a} = \begin{pmatrix}0\\0\\1\end{pmatrix}; \quad \vec{n} = \begin{pmatrix}2\\0\\1\end{pmatrix}$

$\underline{CP}$: $\sphericalangle(\vec{a}, \vec{n}) = \text{angle}\left(\begin{pmatrix}0\\0\\1\end{pmatrix}, \begin{pmatrix}2\\0\\1\end{pmatrix}\right) = 63{,}4°$

$\Rightarrow \quad \sphericalangle(z\text{-Achse}, E) = 90° - 63{,}4° = \underline{\underline{26{,}6°}}$

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b) $\sphericalangle(g, xy\text{-Ebene})$

Gleichung der Geraden g: $\vec{x} = \begin{pmatrix}3\\1\\1\end{pmatrix} + t\begin{pmatrix}2\\1\\0\end{pmatrix}$

Gleichung der xy-Ebene: $z = 0$

$\Rightarrow \quad \vec{a} = \begin{pmatrix}2\\1\\0\end{pmatrix}; \quad \vec{n}_{xy} = \begin{pmatrix}0\\0\\1\end{pmatrix}$

$\underline{CP}$: $\sphericalangle(\vec{a}, \vec{n}_{xy}) = \text{angle}\left(\begin{pmatrix}2\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}\right) = 90°$

$\Rightarrow \quad \sphericalangle(g, xy\text{-Ebene}) = 90° - 90° = \underline{\underline{0°}}$ $g \parallel xyE$

$\sphericalangle(g, xz\text{-Ebene})$

$\Rightarrow \quad \vec{a} = \begin{pmatrix}2\\1\\0\end{pmatrix}; \quad \vec{n}_{xz} = \begin{pmatrix}0\\1\\0\end{pmatrix}$

$\underline{CP}$: $\sphericalangle(\vec{a}, \vec{n}_{xz}) = \text{angle}\left(\begin{pmatrix}2\\1\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}\right) = 63{,}4°$

$\Rightarrow \quad \sphericalangle(g, xz\text{-Ebene}) = 90° - 63{,}4° = \underline{\underline{26{,}6°}}$

$\sphericalangle(g, yz\text{-Ebene})$

$\vec{a} = \begin{pmatrix}2\\1\\0\end{pmatrix}; \quad \vec{n}_{yz} = \begin{pmatrix}1\\0\\0\end{pmatrix}$

$\underline{CP}$: $\sphericalangle(\vec{a}, \vec{n}_{yz}) = \text{angle}\left(\begin{pmatrix}2\\1\\0\end{pmatrix}, \begin{pmatrix}1\\0\\0\end{pmatrix}\right) = 26{,}6°$ $\Rightarrow \sphericalangle(g, yz\text{-E}) = 90° - 26{,}6° = 63{,}4°$

S. 356 / 5

$g: \vec{x} = \begin{pmatrix} 2 \\ 1 \\ -5 \end{pmatrix} + r \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$; $h: \vec{x} = \begin{pmatrix} 3 \\ 2 \\ -5 \end{pmatrix} + r \begin{pmatrix} -2 \\ 3 \\ 7 \end{pmatrix}$

$E: x - 2y + 5z = 7$; $F: y + z = 0$

a) $\measuredangle(g, h)$: $\cos\measuredangle(g,h) = \cos\measuredangle(\vec{a}, \vec{b}) = \dfrac{|\vec{a} \circ \vec{b}|}{|\vec{a}| \cdot |\vec{b}|}$

$\curvearrowright \cos\measuredangle(\vec{a}, \vec{b}) = \dfrac{\left|\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \circ \begin{pmatrix} -2 \\ 3 \\ 7 \end{pmatrix}\right|}{\sqrt{1+1} \cdot \sqrt{4+9+49}} = \dfrac{1}{\sqrt{2} \cdot \sqrt{62}}$

$\measuredangle(\vec{a}, \vec{b}) = \arccos(0{,}0898)$

$\measuredangle(\vec{a}, \vec{b}) = \underline{\underline{84{,}9^\circ}}$

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b) $\measuredangle(E, F) = \underline{\underline{67{,}2^\circ}}$; c) $\measuredangle(g, E) = \underline{\underline{7{,}4^\circ}}$

S. 357 | 10

a) $\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$; $\vec{n} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$; wegen $\vec{a} \circ \vec{n} = 1 - 4 + 3 = 0$

folgt: $\vec{a} \perp \vec{n} \curvearrowright g \parallel E$

$\curvearrowright \measuredangle(g, E) = \underline{\underline{0^\circ}}$

b) $\vec{a} = \begin{pmatrix} 1 \\ 9 \\ 3 \end{pmatrix}$; $\vec{n} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$; wegen $\vec{a} \circ \vec{n} = 3 - 9 + 6 = 0$

folgt $\vec{a} \perp \vec{n} \curvearrowright g \parallel E$

$\curvearrowright \measuredangle(g, E) = \underline{\underline{0^\circ}}$

c) $\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$; $\vec{n} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$; wegen $\vec{a} \circ \vec{n} = 1 + 4 - 9 = -4 \neq 0$

folgt $\measuredangle(g, E) \neq 0^\circ$

$\curvearrowright \measuredangle(\vec{a}, \vec{n}) = \text{angle}\left(\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}\right) = 106{,}6^\circ$

$\curvearrowright \measuredangle(g, E) = |90^\circ - 106{,}6^\circ| = \underline{\underline{+16{,}6^\circ}}$

d) $\measuredangle(\vec{a}, \vec{n}) = \text{angle}\left(\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}\right) = 82{,}8^\circ \curvearrowright \measuredangle(g, E) = \underline{\underline{7{,}2^\circ}}$