$$f(x) = \frac{x^3}{x^2-1}$$

1

1) $D_f = \{x \mid x \neq \pm 1, x \in \mathbb{R}\}$; $P_Y(0|0)$

Punktsymmetrie?: $f(x) = -f(-x)$

$$\frac{x^3}{x^2-1} = -\frac{(-x)^3}{(-x)^2-1}$$

$$= \frac{x^3}{x^2-1} \text{ wahr!}$$

2) Nst: $0 = \frac{x^3}{x^2-1} \quad | \cdot (x^2-1)$

$0 = x^3 \curvearrowright \underline{x_0 = 0}$

3) Unstetigkeiten: Vermutung: bei $x = 1$ und $x = -1$

$\lim\limits_{x \to 1} \frac{x^3}{x^2-1} = \frac{1}{0} \curvearrowright$ Polstelle; $\lim\limits_{x \to -1} \frac{x^3}{x^2-1} = \frac{-1}{0} \curvearrowright$ Polst.

4) lokale Extrema:

$f'(x) = \frac{x^4-3x^2}{(x^2-1)^2} \curvearrowright$ nB: $0 = x^4 - 3x^2 = x^2(x^2-3)$

$\underline{x_{E_1} = 0}$ $\quad$ $\underline{x_{E_{2,3}} = \pm\sqrt{3}}$

$f''(x) = \frac{2x^3+6x}{(x^2-1)^3} \curvearrowright$ h.B: $f''(0) = 0 \curvearrowright ?$

$f''(\sqrt{3}) = \frac{2 \cdot \sqrt{3}^3 + 6 \cdot \sqrt{3}}{8} = \frac{12\sqrt{3}}{8} > 0$

$\curvearrowright \underline{P_T(\sqrt{3}|2{,}6)}$

$f''(-\sqrt{3}) = \frac{-12\sqrt{3}}{8} < 0$

$\curvearrowright \underline{P_H(-\sqrt{3}|-2{,}6)}$

5) Wendepunkte:

$f''(x) = \frac{2x^3+6x}{(x^2-1)^3} \curvearrowright$ nB: $0 = 2x^3 + 6x = x(2x^2+6)$

$\underline{x_{W_1} = 0}$ $\quad$ $x_{W_{2,3}} = $ n.l.

$f'''(x) = \frac{-6x^4-36x^2-6}{(x^2-1)^4} \curvearrowright$ h.B: $f'''(0) = -6 < 0 \curvearrowright$ LiReKu

$\curvearrowright \underline{P_W(0|0)}$

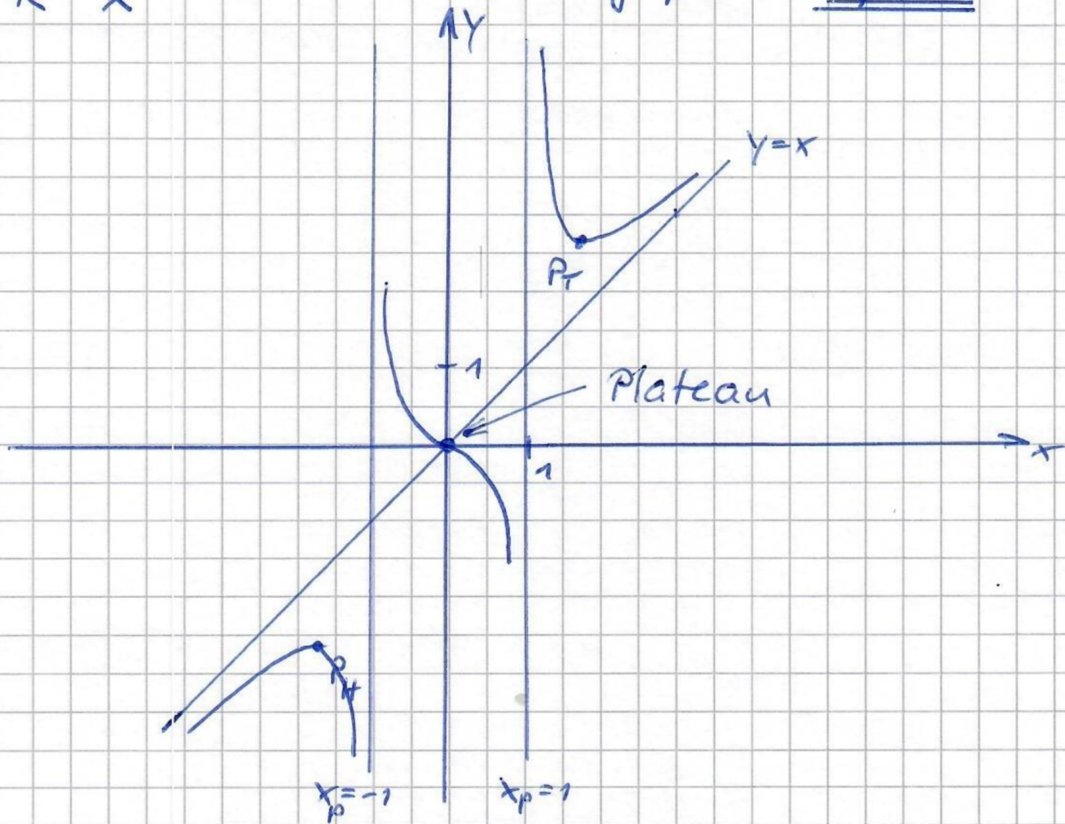
6) $x \to \pm\infty$

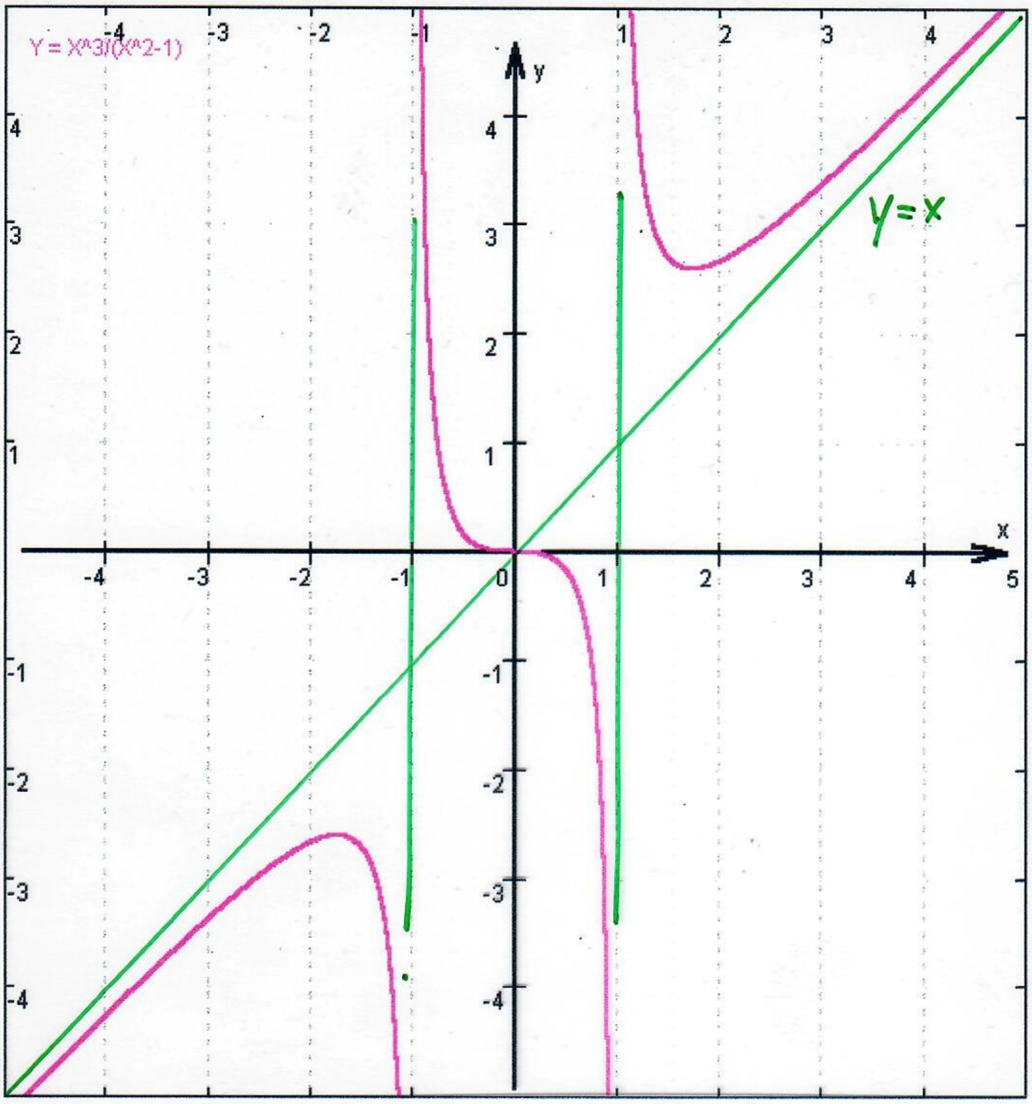
$$\lim_{x \to \pm\infty} \frac{x^3}{x^2-1} = \lim_{x \to \pm\infty} \frac{\cancel{x^3}\, x}{\underbrace{\cancel{x^2}\left(1-\frac{1}{x^2}\right)}_{1}} = \pm\infty$$

$$(x^3) : (x^2-1) = \underline{\underline{x}} + \frac{x}{x^2-1}$$
$$\frac{-(x^3-x)}{R \quad x}$$

Asymptote: $\underline{\underline{y = x}}$

7)




Y = X^3/(X^2-1)
y
x
Y=x